Discussion of Dong and Wen 'Long and Plosser Meet Bewley and Lucas'

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Long and Plosser

Long and Plosser (1983) 'Real Business Cycles' *Journal of Political Economy* **91**(1), pages 39 - 69.

The first N-sector Real Business Cycle Model. An important role for sectoral shocks in aggregate fluctuations.

- Perfectly Competitive Firms produce using constant returns to scale technologies subject to technology shocks
- Sectoral output serves as both consumption and an intermediate input
- One-period time to build for intermediate inputs, full depreciation
- Hours worked in each sector pre-determined

Real Business Cycle Model

$$Y_i \leq \lambda_i L_i^{b_i} \prod_{j=1}^{N} S_{i,j}^{a_i},\ i = 1, \ldots, N. \tag{1}$$

$S_{i,j}$ are intermediate inputs, output from sector j, and λ_i is a technology shock

$$C_i + \sum_{j=1}^{N} S'_{j,i} \leq Y_i \tag{2}$$

C_i consumption of good i. Let $C = \{C_i\}_{i=1}^{N}$, $L = \{L_i\}_{i=1}^{N}$ and $\lambda = \{\lambda_i\}_{i=1}^{N}$.

$$\begin{aligned} V(S, \lambda) &= \max_{C, L, S'} \left(u(C, L) + \beta \int V\left(S', \lambda'\right) Q\left(\lambda, d\lambda'\right) \right) \\ &\quad \text{subject to (1) and (2).} \end{aligned} \tag{3}$$

In this version, hours worked chosen contemporaneously.

$$u(C, L) = \sum_{i=1}^{N} \varphi_i \log C_i - \sum_{i=1}^{N} L_i$$

Long and Plosser Model

analytical solution

Let γ_i be the state-invariant marginal propensity to consume good i.

$$C_i = \gamma_i Y_i,\ i = 1, \ldots, N.$$

Define $x = \left(\gamma_1^{-1} \ldots \gamma_N^{-1}\right)'$ and $b = (\varphi_1 \ldots \varphi_N)'$. Let

$$A = \begin{vmatrix} -\varphi_1 + \beta a_{1,1}\varphi_1 & \beta a_{2,1}\varphi_2 & \cdots & \beta a_{N,1}\varphi_N \\ \beta a_{1,2}\varphi_1 & \beta a_{2,2}\varphi_2 & \cdots & \beta a_{N,2}\varphi_N \\ \vdots & \vdots & & \vdots \\ \beta a_{1,N}\varphi_1 & \beta a_{2,N}\varphi_2 & \cdots & \beta a_{N,N}\varphi_N \end{vmatrix}$$

Then $x = A \backslash b$ solves for $(\gamma_1, \ldots, \gamma_N)$ and, over $i = 1, \ldots, N$,

$$L_i = \frac{\varphi_i}{\gamma_i} b_i \quad \text{and} \quad S'_{i,j} = \beta a_{i,j} \frac{\varphi_i}{\gamma_i} \frac{\gamma_j}{\varphi_j} Y_j,\ j = 1, \ldots, N.$$

Hours worked is constant as are investment rates, $\frac{S'_{i,j}}{Y_j}$.

Dong and Wen

Introduce uninsurable individual risk and money into the Long and Plosser model.

Households hold money to smooth consumption (Bewley 1983).

Unanticipated money growth shocks have persistent real effects.

Low marginal propensity to consume households increase real balances.

Velocity falls, GDP increases.

Sectoral Business Cycles with Real and Nominal Shocks

Introduce preference shocks (Lucas 1980), common across all goods for a household; i.i.d. across households and over time.

$$u\left(C,L\right)=\theta\sum_{i=1}^{N}\varphi_{i}\log c_{i}-\sum_{i=1}^{N}l_{i}.$$

Households must choose hours worked, $\left(l_{j}\right)_{j=1}^{N}$, and future real assets, $\left(s'_{i,j}\right)_{i,j=1}^{N}$, before observing $\theta\sim F\left(\theta\right)$.

This leaves them with *cash-on-hand*, x, for consumption $\left(c_{j}\right)$ or money savings, m.

$$\sum_{j=1}^{N}q_{j}c_{j}+m\leq x,$$

q_{j} is the relative price of good j.

Household start-of-period

$$V\left(m_{-1},\left(s_{i,j}\right)_{i,j=1}^{N};\left(\lambda_i\right)_{i=1}^{N},\gamma,K\right)=\max_{x,\left(l_j\right)_{j=1}^{N},\left(s'_{i,j}\right)_{i,j=1}^{N}}\left(-\sum_{j=1}^{N}l_j\right.$$

$$\left.+\int J\left(\theta,x,\left(s'_{i,j}\right)_{i,j=1}^{N};\left(\lambda_i\right)_{i=1}^{N},\gamma,K\right)F\left(d\theta\right)\right)$$

subject to

$$x=\left(m_{-1}+\gamma\overline{m}_{-1}\right)\frac{P_{-1}}{P}+\sum_{j=1}^{N}q_j\widetilde{y}_j-\sum_{j=1}^{N}\sum_{i=1}^{N}q_j s'_{i,j}$$

$$\widetilde{y}_j=\sum_{i=1}^{N}\left(1+r_{j,i}\right)s_{j,i}+w_j l_j,\ j=1,\ldots,N.$$

$r_{i,j}$ is the real rate of return, in units of good i on $s_{i,j}$.

w_j is the real wage, also in units of good j.

$\gamma\overline{m}_{-1}$ is a lump-sum money transfer and P is the price level.

Household after preference shock

$$J\left(\theta, x, \left(s'_{i,j}\right)_{i,j=1}^{N}; \left(\lambda_i\right)_{i=1}^{N}, \gamma, K\right) = \max_{\left(c_j\right)_{j=1}^{N},\ m'} \left(\theta \sum_{j=1}^{N} \varphi_j \log c_j + \int V\left(m, \left(s'_{i,j}\right)_{i,j=1}^{N}; \left(\lambda'_i\right)_{i=1}^{N}, \gamma', K'\right) Q\left(\lambda, d\lambda'\right) R\left(\gamma, d\gamma'\right)\right)$$

subject to

$$\sum_{j=1}^{N} q_j c_j + m \leq x,$$

$$K' = \Gamma\left(\lambda, \gamma, K\right).$$

Market Incompleteness

consumption and real balances

Constant marginal utility of leisure eliminates persistent heterogeneity.

At the start of a period, household hold identical portfolios of real assets, $s(i,j)$.

Wealth varies only in real balances. Households that experienced high θ_{-1} consumed more of their x_{-1} and reduced m_{-1}.

$$c_j(h) = \frac{\varphi_j}{q_j} \min\left\{1, \frac{\theta(h)}{\theta^*}\right\} x$$

$$m(h) = \max\left\{1 - \frac{\theta(h)}{\theta^*}, 0\right\} x$$

The preference shock θ is i.i.d., drawn from a Pareto (type 1) distribution with mean 1 and shape parameter, η.

Individual Consumption and Earnings

Differences in real balances are immediately offset by variation in hours worked.

$$\begin{aligned}
\sum_{j=1}^{N} q_j \widetilde{y}_j(h) &= x + \sum_{j=1}^{N}\sum_{i=1}^{N} q_j s'_{i,j}\gamma + \frac{P_{-1}}{P}\overline{m}_{-1} - \frac{P_{-1}}{P} m_{-1}(h) \\
m_{-1}(h) &= \max\left\{1 - \frac{\theta_{-1}(h)}{\theta^*}, 0\right\} x, \\
\sum_{j=1}^{N} q_j \widetilde{y}_j(h) &= \sum_{i=1}^{N} \left(1 + r_{j,i}\right) s_{j,i} + w_j l_j(h).
\end{aligned}$$

Income varies, poorer households consumed more last period, and work harder.

As θ is i.i.d. for any household over time, and the household-specific determinants of hours worked involve offsetting the effect of volatility in start of period real balances, there is no correlation between *current* income and consumption.

Individual consumption and income are both volatile, but neither has persistence.

Cross-sectional distribution of consumption and income

Consumption

	Q1	Q2	Q3	Q4	Q5	Gini
CEX (1995-2005)	9	12	16	22	41	32
model	9.9	11.9	14.9	21.6	41.6	31

$\eta = 2.5$ fits the cross-sectional distribution of consumption well

Income

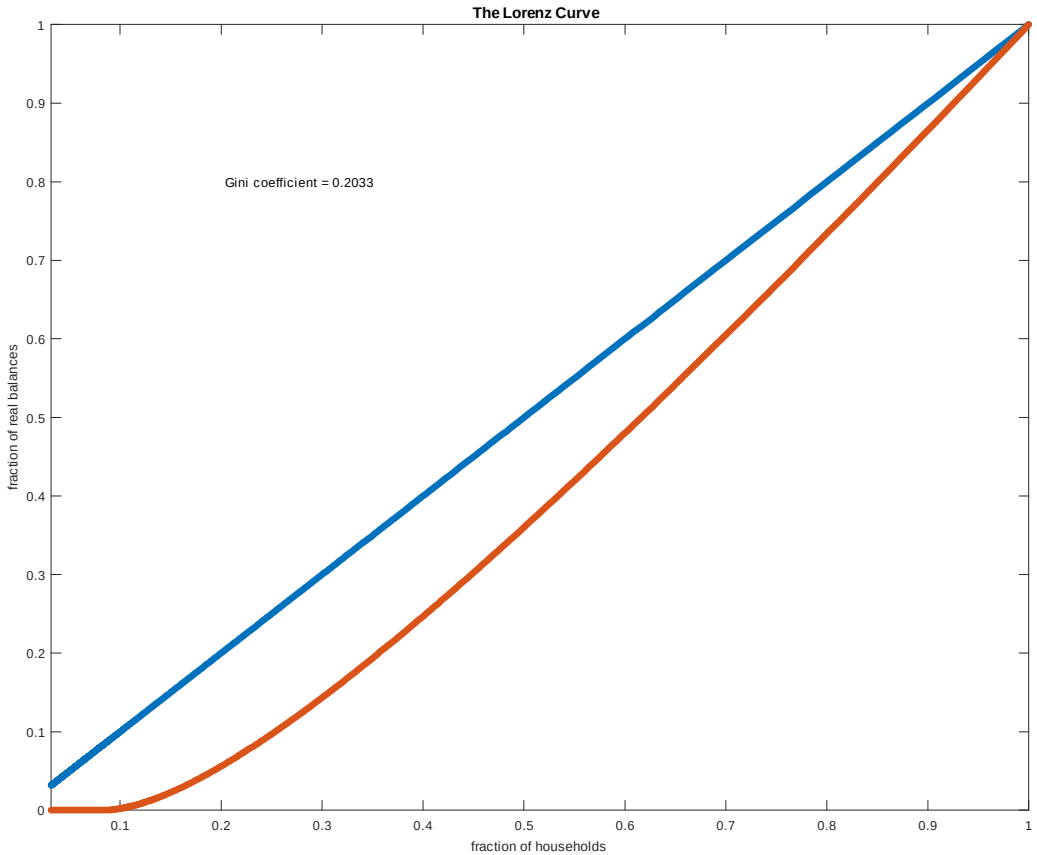
	Q1	Q2	Q3	Q4	Q5	Gini
CEX (1995-2005)	6	11	16	23	43	37
model	18.1	18.6	19.2	20.5	23.6	5

Income is more unequal than consumption in the data.

Little income inequality in model suggests *high individual consumption risk*.

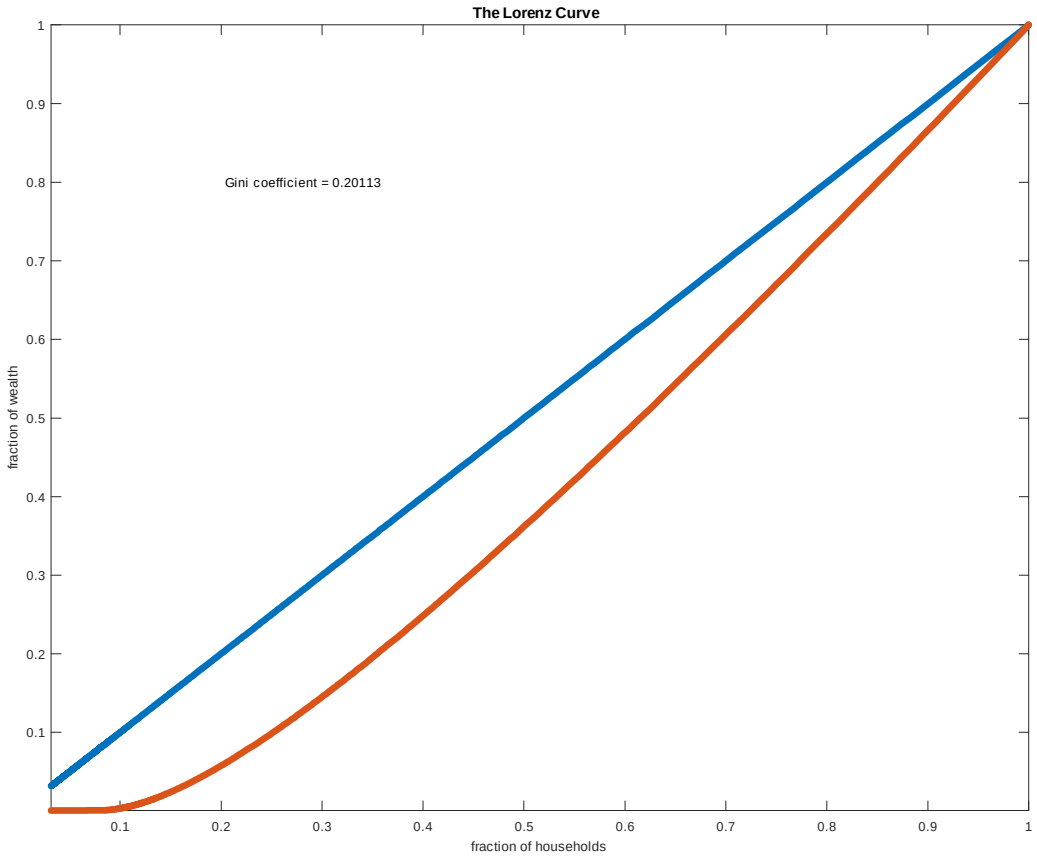
This raises x and real balances, relative to a model with less risk.

Real balances distribution



Roughly 8 percent of households spend all their real balances because $\theta > \theta^*$.

Wealth distribution



The wealth gini is 0.2 in the model, but around 0.72 in trimmed SCF (Heathcote, Perri and Violante 2010).

Input-Output Table

Structural Changes in Sectoral Trade

Long and Plosser (1983)

	agriculture	mining	construction	manufacturing	trans. and trade	services
agriculture	0.4471	0.0033	0.0146	0.2093	0.0999	0.1591
mining	0.0000	0.0935	0.0427	0.1744	0.0549	0.4854
construction	0.0029	0.0104	0.0003	0.4189	0.1209	0.0893
manufacturing	0.0618	0.0340	0.0050	0.4576	0.0611	0.1267
trans. and trade	0.0017	0.0004	0.0166	0.1246	0.1040	0.3249
services/ others	0.0174	0.0212	0.0595	0.1998	0.0871	0.3805

2007 Data

	agriculture	mining	construction	manufacturing	trans. and trade	services
agriculture	0.2894	0.0083	0.0300	0.2823	0.1294	0.0897
mining	0.0005	0.2548	0.0566	0.1691	0.0690	0.1826
construction	0.0012	0.0635	0.0117	0.2903	0.1328	0.1045
manufacturing	0.0477	0.0981	0.0219	0.4340	0.0915	0.1130
trans. and trade	0.0004	0.0020	0.0161	0.0890	0.1124	0.3165
services/ others	0.0008	0.0024	0.0309	0.0792	0.0284	0.3405

Real effects of persistent money growth shock

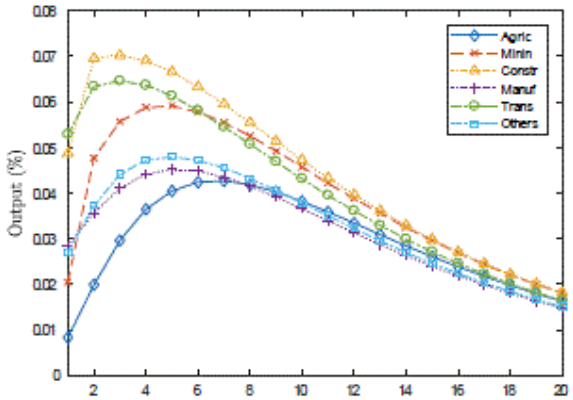


Figure 6a. Impulse response of sectoral output to monetary shock.

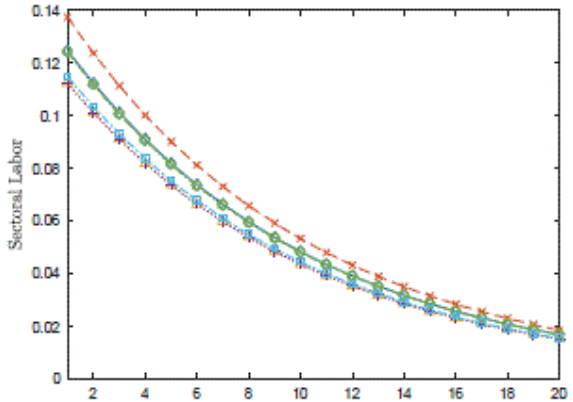


Figure 6b. Impulse responses of sectoral labor to monetary shock.

A positive money growth shock increases θ^* reducing the fraction of households that are liquidity constrained.

Mechanics of a Money Growth Shock

Velocity and the distribution of marginal propensity to consume and

An unanticipated shock to the money supply; lump-sum transfers to households.

Households with $\theta \leq \theta^*$ have a *low marginal propensity to consume*, they increase real balances.

Nominal spending rises by less than money supply; *velocity falls*.

Production increases, future income rises.

These effects seem small, given the high value of θ^*, the number of liquidity constrained households is 8.1 percent.

Similar mechanics to Segmented Markets Models: Alvarez, Atkeson and Edmond (2009), Khan and Thomas (2015)

Implications for money supply shocks

role of consumption risk in money demand

If I increase η to 2.65, consumption Gini falls to 28 from 31.

The variance of log consumption falls from 26.1 to 21.9 percent.

The variance of log income falls from 7.3 to 4.9 percent.

Real balances fall 77 percent, and the aggregate response in consumption to a persistent money growth shock declines by 1/3.

Concluding remarks

Money in the Long and Plosser model

A challenge has been to maintain the importance of sectoral shocks, for aggregate fluctuations, with greater disaggregation.

Dong and Wen do not directly address this issue.

Instead they extend the Long and Plosser model to allow for monetary shocks.

Qualitatively interesting though quantitative effects are small.

An important future direction would amplify the effect of nominal shocks.

However, if the variance of preference shocks were calibrated using panel data on individual consumption and earnings, it should fall.

A reduction in individual consumption volatility would reduce the demand for real balances and perhaps the real effects of nominal shocks.